

Indian Institute of Technology Dharwad
Logic and applications
Assignment 1
Due date: 8 September 2023

Theorem 1 (Finite Models Theorem(FMT)). *If $\Sigma \models \alpha$ then $\exists \Sigma_0 \subseteq_{fin} \Sigma$ and $\Sigma_0 \models \alpha$.*

Theorem 2 (Compactness Theorem(CT)). *Σ is satisfiable iff Σ is finitely satisfiable.*

1. Formalize the following statements as formulas, and then show that they are equivalent. (5 marks)
 - (a) If the child has fever or has a bad cough and we reach the doctor over phone, then we ask him to come home.
 - (b) If the child has fever, then we ask the doctor to come home provided we reach him over the phone and, if we reach the doctor over phone then we ask him to come home, if the child has a bad cough.
 2. Give an example of 3-element set M of wffs, such that M is not satisfiable, but every 2-element subset of M is satisfiable. Generalize your example to n element sets. (5 marks)
 3. Prove the following: Theorem 1 if and only if Theorem 2. (5 marks)
 4. Prove the following statement :
Any set of wffs Σ is maximally satisfiable if and only if there is exactly one assignment satisfying Σ . (5 marks)
 5. Show that $\{\neg, \iff, \oplus\}$ is not adequate.
Here \oplus is XOR function. (5 marks)
 6. A graph G is 3-colorable if and only if every finite subgraph G' of G is 3-colorable. (5 marks)
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