

Indian Institute of Technology Dharwad

Quiz: CS 621 Logic and Applications

Total marks : 5*4=20

Time:50 min

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1. If F is formula, and F occurs as part of the formula G , then F is called a *subformula* of G . A formula without occurrence of a free variable is called *closed*.

Consider the following formula:

$$\phi = (Q(x) \vee (\exists x \forall y (P(f(x), z) \wedge Q(a))) \vee \forall x R(x, z, g(x)))$$

List all subformulas and terms that occur in the formula. Which subformulas are closed? Determine for each occurrence of a variable if it is free or bound.

2. Suppose $F(x)$ means "x is a person", $G(x)$ means "x is a time" and $H(x,y)$ means "one can fool x at y".
Translate (if the proposition is ambiguous, you will need more than one translation):
 - (a) One can fool some of the people at all of the time.
 - (b) One can fool all of the people at some of the time.
 - (c) One can not fool all of the people at all of the time.
3. Consider the following formula:

$$\phi = \forall x \exists y P(x, y, f(z)).$$

Here, P is a relational symbol and f is a function. Define a suitable structure which is a model for ϕ , and define another suitable structure which is not a model for ϕ .

4. Prove or disprove the following:
 - (a) $\forall x \neg A(x) \models \neg \exists x A(x)$.
 - (b) $\forall x (A(x) \implies B(x)) \models (\forall x A(x) \implies \forall x B(x))$