CS621: Logic and applications

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Z3 SAT solver and encoding

Prerequisite

Z3 installed on your machines?

`https://github.com/Z3Prover/z3`

SAT solver: satisfiability solver (tool)

Solver Interface

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- Output: sat/unsat/unknown

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What if the solver returns unknown? timed out

A SAT solver: Z3

- Z3 is a state of the art theorem prover/ SAT solver built by Microsoft Research
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Which API? I will use Z3 cpp for today

A SAT solver: Z3 contd

Lets get familiar with Z3 syntax

A SAT solver: Z3 contd

- Step 1: Constructing the input formula (encoding the puzzle into a formula)

A SAT solver: Z3 contd

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  - expr
  - expr_vector

A SAT solver: Z3 contd

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- Step 2: Adding the formula to the solver instance
  - solver instance
  - context instance
  - The context consists of a formula store containing the constructed formulas

A SAT solver: Z3 contd

- Step 1: Constructing the input formula (encoding the puzzle into a formula)
- Step 2: Adding the formula to the solver instance
  - solver instance
  - context instance
  - The context consists of a formula store containing the constructed formulas
  - Z3 Python interface instantiates a default context (implicitly)

A SAT solver: Z3 contd

- Step 1: Constructing the input formula (encoding the puzzle into a formula)
- Step 2: Adding the formula to the solver instance
  - *solver*.*add*(*formula*)
  - Solver solves only the added formula

A SAT solver: Z3 contd

- Step 1: Constructing the input formula (encoding the puzzle into a formula)
- Step 2: Adding the formula to the solver instance
  - $solver.add(formula)$
- Step 3: $solver.check()$

Encoding a simple propositional logic formula

Prove the following

$\neg(x \wedge y) = \neg x \vee \neg y$

[1] g++ demorgan.cpp -lz3

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What is the expected outcome? sat

Lets see how to encode this.

1

[1] g++ demorgan.cpp -lz3

Any questions so far?

Encoding problems: solving a set of constraints

How to find values of expressions that satisfy the following

$x \geq 1$

$y < x + 3$

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How is this written in smt2?

Encoding problems: solving a set of constraints

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$x \geq 1$

$y < x + 3$

What is the expected outcome? sat

Lets see a demonstration

How is this written in smt2? **Fetching a satisfying model?**

Encoding the sudoku puzzle

	2		5		1		9	
8			2		3			6
	3			6			7	
		1				6		
5	4						1	9
		2				7		
	9			3			8	
2			8		4			7
	1		9		7		6	

4	2	*6*	5	*7*	1	*3*	9	*8*
8	*5*	*7*	2	*9*	3	*1*	*4*	6
1	3	*9*	*4*	6	*8*	*2*	7	*5*
9	*7*	1	*3*	*8*	*5*	6	*2*	*4*
5	4	*3*	*7*	*2*	*6*	*8*	1	9
6	*8*	2	*1*	*4*	*9*	7	*5*	*3*
7	9	*4*	*6*	3	*2*	*5*	8	*1*
2	*6*	*5*	8	*1*	4	*9*	*3*	7
3	1	*8*	9	*5*	7	*4*	6	*2*

Encoding problems: sudoku

- How do you encode sudoku puzzle?

Encoding problems: sudoku

- How do you encode sudoku puzzle?
- What is the input?

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```
for ( i = 0; i < 9; ++i)
for ( j = 0; j < 9; ++j) {
solver.add(x[i * 9 + j] ≥ 1 && x[i * 9 + j] ≥ 9);
}
```

Is this correct?

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Encoding problems: sudoku

- What is the input? A partially filled matrix say x
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  1. Each cell contains a value in 1,. . . ,9
  2. Each row contains a digit at most once

```
for ( i = 0; i < 9; ++i) {
expr_vector t(c);
for (j = 0; j < 9; ++j)
t.push_back(x[i * 9 + j]);
s.add(distinct(t));
}
```

Encoding problems: sudoku

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- What are my constraints?
  1. Each cell contains a value in 1,. . . ,9
  2. Each row contains a digit at most once
  3. Each column contains a digit at most once

Encoding problems: sudoku

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- What are my constraints?
  1. Each cell contains a value in 1,. . . ,9
  2. Each row contains a digit at most once
  3. Each column contains a digit at most once
  4. Each 3x3 square block contains a digit at most once

- An assignment by this weekend.
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Thank you