Calculation of Mixed Boundary Conditions

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1 Residuum Method

Make step of $\partial \boldsymbol{P}$ from current situation towards $\boldsymbol{P}_{wish}$ (leaving free components as they are). This results in $\mathrm{d}\boldsymbol{F} = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{P}} \cdot \mathrm{d}\boldsymbol{P}$ which has potentially non-zero components at prescribed F_{ij}. Isolate those as residuum $\boldsymbol{R}$. Use $\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{F}} \cdot \boldsymbol{R}$ to calculate change of $\mathrm{d}\boldsymbol{P}$ to kill $\boldsymbol{R}$. Adjust initial $\mathrm{d}\boldsymbol{P}$ by change and calculate $\mathrm{d}\boldsymbol{F} = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{P}} \cdot \mathrm{d}\boldsymbol{P}_{corr}$ which should be now free of fixed components. Check wether $\mathrm{d}\boldsymbol{F} = \boldsymbol{R} \cdot \boldsymbol{U}$ has $\boldsymbol{R} \approx \boldsymbol{I}$

2 Reduction Method

Calculate $\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{P}}$ and convert into 9x9 matrix. Convert $\boldsymbol{\Delta P}$ and $\boldsymbol{\Delta F}$ into an vector of dimension 9. Remove all entries from the stiffness matrix where $\boldsymbol{F}$ is prescribed (and where $\boldsymbol{\Delta P}$ is undefined). Invert the reduced matrix and fill the removed entries with 0. For a linear behavior, $\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{P}}_{red} : \boldsymbol{\Delta P}$ gives the exact change of $\boldsymbol{\Delta F}$.